Algebra Homework 5

Due by the *start* of class on Wednesday Nov. 4

Problem 1: Find two different field extensions of $\mathbb{Z}_3$, one with 9 elements and another with 27 elements, in which the polynomial $f(x) = x^5 + 2x^2 + 2x + 2 \in \mathbb{Z}_3[x]$ has a root (hint: one factor of $f(x)$ is $x^2 + 1$).

Problem 2: Draw steps for how to bisect an angle using a straightedge and compass. You shouldn't need more than two or three pictures, and you don't have to "prove" that your construction works – a convincing picture is enough.

Problem 3: Prove that if a regular n-gon is constructible, then a regular $2n$-gon is also constructible. Your "proof" should consist of a precise description of how to construct the $2n$-gon.

Problem 4: Prove that $\pi^2 - 1$ is algebraic over $\mathbb{Q}(\pi)$.

Problem 5: Prove that $\mathbb{Q}(\sqrt{3}, \sqrt[3]{3}, \sqrt[4]{3}, \dots)$ is an algebraic field extension of $\mathbb{Q}$, but that it is *not* a finite extension.