Algebra Homework 4

Due by the *start* of class on Wednesday Oct. 28

Problem 1: Show that $\mathbb{Q}(4-i) = \mathbb{Q}(1+i)$, where $i = \sqrt{-1} \in \mathbb{C}$.

Problem 2: Let $p \in \mathbb{Z}$ be any prime number, and $n \geq 2$

1. Find a basis for $\mathbb{Q}(\sqrt[n]{p})$ as a vector space over $\mathbb{Q}$ (justify your answer!)
2. Calculate $[\mathbb{Q}(\sqrt[n]{p}) : \mathbb{Q}]$.
3. Find an example that shows that your answer to part (2) might not be true if p is not prime.

Problem 3: Let $\alpha = \sqrt{3+\sqrt{6}}$.

1. Find a polynomial in $\mathbb{Q}[x]$ which has α as a root.
2. What is a basis for $\mathbb{Q}(\alpha)$ as a vector space over $\mathbb{Q}$ (justify your answer!)
3. Calculate $[\mathbb{Q}(\alpha) : \mathbb{Q}]$.

Problem 4: Let ξ be the complex number $e^{\frac{\pi i}{3}}$.

1. Calculate a basis for each of $\mathbb{Q}(\xi^1), \mathbb{Q}(\xi^2)$, and $\mathbb{Q}(\xi^3)$ as a vector space over $\mathbb{Q}$ (justify your answers!).
2. Calculate $[\mathbb{Q}(\xi^1) : \mathbb{Q}], [\mathbb{Q}(\xi^2) : \mathbb{Q}]$, and $[\mathbb{Q}(\xi^3) : \mathbb{Q}]$.

Problem 5: Prove that if $[F(\alpha) : F]$ is odd, then $F(\alpha^2) = F(\alpha)$.