Name: ____________________________

Student Number: ____________________________

Exam 2

Math 401, Fall 2015, Geoffrey Scott

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

Instructions:

- Put your name and student number on this page.
- You have fifty minutes to complete the exam. We will write the remaining time on the blackboard. I recommend planning to spend 10 minutes on each question, which leaves an extra 10 minutes for the question you get stuck on.
- You may ask Nikita or me questions, but we cannot answer math questions or questions like "have I shown enough work?"
- A "dictionary of ring theory" is not on the next page.
- I will give partial credit for some questions, so show your work.
- You may leave early.

Grades:

Question 1: ________ (out of 12)

Question 2: ________ (out of 10)

Question 3: ________ (out of 8)

Question 4: ________ (out of 8)

Total: ________ (out of 38)

Question 1: Short Answer

For the following questions, either give an example of the object described, or explain in one or two sentences why an example cannot exist.

1. An ideal in $\mathbb{Q}(\sqrt{2}, \sqrt{3})[x]$ which is not principal.(4 points)

2. A reducible polynomial $f \in \mathbb{Q}[x]$ with degree ≥ 1 with the property that $f(a) \neq 0$ for all $a \in \mathbb{Q}$.(4 points)

3. A field K with the property that $\mathbb{Q} \subsetneq K \subsetneq \mathbb{Q}(\sqrt[7]{3})$. (4 points)

Question 2: Adjunction

1. Let $\alpha = \sqrt[3]{2+\sqrt{2}}$. Find an irreducible polynomial in $\mathbb{Q}[x]$ that has α as a root. Explain (in one sentence) how you know that the polynomial is irreducible. (4 points)

2. Calculate $[\mathbb{Q}(\alpha) : \mathbb{Q}]$. You do not need to show your work. (2 points)

3. Let n be your answer to Part 2. Write α^{n+2} as a linear combination of $\{1, \alpha, \dots, \alpha^{n-1}\}$ with coefficients in $\mathbb{Q}$. (4 points)

Question 3: Algebraic and Transcendental elements

Let F be a field and let f be any polynomial of degree ≥ 1 in $F[x]$. Let α be an element of an extension field of F such that α is transcendental over F, and let $\beta = f(\alpha)$.

1. Prove that α is algebraic over the field $F(\beta)$. (4 points)

2. Prove that β is transcendental over F. You may use Part 1. (i.e. you may assume that α is algebraic over $F(\beta)$ even if you personally failed to prove it) (4 points)

Question 4: Geometric Constructions

Suppose the points $(0,0)$ and $(0,1)$ are drawn on the plane $\mathbb{R}^2$. For the following two questions, construct the objects described using straightedge and compass constructions from class, or explain why it is not possible.

To show a construction, give a sequence of pictures illustrating the steps (you can do more than one step per picture, as long as the sequence is clear from the pictures). You may use "complicated" operations that we proved were possible in class or in homework. You may use the back of this page to draw constructions if you need more space.

1. Two lines which are distance π apart. (4 points)

2. A right triangle with side lengths $1, \sqrt{3}, 2$ (4 points)